

Discrete symmetries and neutrino masses

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Abstract

We constructed a model of neutrino masses using Froggatt-Nielsen mechanism with $U(1) \times Z_3 \times Z_2$ flavor symmetry. The model predicts that $(2/3)m_2/m_3 \sim \sqrt{2} \sin \theta_{13}$ at lepton number violating scale M_1 . It is shown that the small values for m_2/m_3 and $\sin \theta_{13}$ are consequences of breaking discrete symmetries.

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I. INTRODUCTION

Having experimental data at our hands, it is undeniable that two of three mixing angles in the lepton sector are large and the other is small. A few detailed types of mixing angle sets depending on theories to derive them are still debatable. Interpreting the atmospheric and solar neutrino experiments [1][2] in terms of two-flavor mixing, the mixing angle for the oscillation of atmospheric neutrinos is understood to be maximal or nearly maximal: $\sin^2 2\theta_{atm} \simeq 1$, whereas the one for the oscillation of solar neutrinos is not maximal but large: $\sin^2 \theta_{sol} \simeq 0.3$. The upper bound $\sin \theta_{reac} \lesssim 0.2$ was obtained from the non-observation of the disappearance of $\bar{\nu}_e$ in the Chooz experiment[3]. The masses of charged leptons are the most precisely measured parameters of the fermions. The data reads $m_e = 0.51 MeV, m_\mu = 106 MeV, m_\tau = 1780 MeV$. Meanwhile, as for neutrinos, the existing data just show that the neutrino mass squared differences which induce the solar and atmospheric neutrino oscillations are $\Delta m_{sol}^2 \simeq (7_{-2}^{+10}) \times 10^{-5} \text{ eV}^2$ and $\Delta m_{atm}^2 \simeq (2.5_{-0.9}^{+1.4}) \times 10^{-3} \text{ eV}^2$, respectively. With SNO[2] and KamLAND[4], data have narrowed down the possible mass spectrum of neutrinos into two types, normal hierarchy ($m_1 \lesssim m_2 < m_3$) and inverse hierarchy ($m_3 < m_1 \lesssim m_2$). If the experimental results $\Delta m_{sol}^2 = m_2^2 - m_1^2$ and $\Delta m_{atm}^2 = |m_3^2 - m_2^2|$ are accommodated to the masses of normal hierarchy, one can obtain the following relations for mass ratio,

$$\frac{m_2}{m_3} \approx \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} = (1.7_{-0.6}^{+1.7}) \times 10^{-1}, \quad (1)$$

assuming m_1 is strongly restricted to be smaller than m_2 by the order of magnitude, rather than $m_1 \lesssim m_2$.

The unitary mixing matrix is defined via $\nu_a = \sum_{j=1}^3 U_{aj} \nu_j$ ($a = e, \mu, \tau$), where ν_a is a flavor eigenstate and ν_j is a mass eigenstate. Including data from SNO[2] and KamLand[4], the range of the magnitude of the MNS mixing matrix is given by [5][6],

$$|U| = \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0 - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix} \quad (2)$$

at the 90% confidence level. It can be readily recognized that the central values of elements in the mixing matrix in Eq.(2) are pointing likely numbers, $\sin \theta_{sol} = \frac{1}{\sqrt{3}}$ and $\sin \theta_{atm} = \frac{1}{\sqrt{2}}$

[7]. If neutrino mixing matrix is close to the lepton mixing MNS matrix given with the values dictated by experiments as above, the neutrino mass matrix in that basis will imbed two leading terms as

$$\mathcal{O}(m_3) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mathcal{O}(m_2) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \mathcal{O}(m_1, s_{13}), \quad (3)$$

if the type of mass spectrum is normal hierarchy. The model to be presented will explain how those different scales of elements in the mass matrix are derived from a certain flavor symmetry.

Froggatt-Nielsen(FN) mechanism[8] with a $U(1)$ flavor symmetry is a commonly used strategy to construct the model of Yukawa interaction for fermions. However, if the FN mechanism with $U(1)$ is taken to build neutrino Yukawa interaction and then the seesaw mechanism[9] completes the construction for the light neutrinos, the mass matrix can not avoid an anarchy type[10],

$$\mathcal{O}(m_3) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (4)$$

since breaking continuous $U(1)$ flavor symmetry alone is capable of generating no gaps in scales of neutrino matrix elements. Unless the approach includes somehow particular strategy, e.g., an accidental fine tuning, the mechanism does not give rise to an explanation of hierarchy in masses and the two large angles and small reactor mixing angle θ_{e3} . The suppressed couplings of ν_e for the natural smallness of θ_{e3} can be constructed using a $U(1)$ flavor symmetry in low-energy effective theories[11]. However the 1-1 element is further suppressed relative to other couplings with ν_e so that the desired structure in Eq.(3) would not be obtained under a $U(1)$ flavor symmetry. We will use an idea which is that, by extending the $U(1)$ flavor symmetry so that it contains additional discrete abelian Z_n symmetries, it is possible to achieve relative suppression or relative enhancement among elements in Yukawa matrices or in neutrino mass matrix[12][13][14]. It will be shown that a flavor symmetry $U(1) \times Z_3 \times Z_2$ originate the particular pattern in Eq.(3).

The model in consideration will be constructed on the basis where the neutrino masses involve two large lepton mixing angles. The neutrino mass matrix is derived via the seesaw

mechanism by introducing three heavy right-handed neutrinos.

In the following section, experimental data will be embedded inside the mass matrix, for comparison with prediction. The elements of mass matrix appear classified scale by scale. In Section III, we adopt the Froggatt-Nielsen(FN) mechanism generating different orders of couplings to establish the structure of Yukawa matrices. We will show that Yukawa couplings generated by breaking discrete abelian flavor symmetries, accompanied with the seesaw mechanism, can produce the distinct scales as in the mass matrix shown as constraint in the previous section. The prediction $(2/3)m_2/m_3 \sim \sqrt{2} \sin \theta_{13}$ will be presented in Section IV. One of the significant outcomes of the model is that the small mass ratio m_2/m_3 and the small mixing U_{e3} are resulted from breaking of the discrete symmetries Z_3 and Z_2 . Some relevant remarks will follow in the last section.

II. LOW ENERGY CONSTRAINTS

In general, a unitary mixing matrix for 3 generations of neutrinos is given by

$$\tilde{U} = R(\theta_{23}) R(\theta_{13}, \delta) R(\theta_{12}) P(\varphi, \varphi') \quad (5)$$

where R 's are rotations with three angles and a Dirac phase δ and the $P = \text{Diag}(1, e^{i\varphi/2}, e^{i\varphi'/2})$ with Majorana phases φ and φ' is a diagonal phase transformation. The mass matrix of light neutrinos is given by $M_\nu = \tilde{U} \text{Diag}(m_1, m_2, m_3) \tilde{U}^T$, where m_1, m_2, m_3 are real positive masses of light neutrinos. Or the Majorana phases can be embedded in the diagonal mass matrix such that

$$M_\nu = U \text{Diag}(m_1, \tilde{m}_2, \tilde{m}_3) U^T, \quad (6)$$

where $U \equiv \tilde{U} P^{-1}$ and $\tilde{m}_2 \equiv m_2 e^{i\varphi}$ and $\tilde{m}_3 \equiv m_3 e^{i\varphi'}$. If the transformation angles are given such that $s_{12} = \frac{1}{\sqrt{3}}(1 + \sigma)$, $s_{23} = \frac{1}{\sqrt{2}}$, $s_{13} \ll 1$, where s_{ij} denotes $\sin \theta_{ij}$ with the mixing angle θ_{ij} between i -th and j -th generations and σ is also small ($\ll 1$), the matrix M_ν can be expressed as

$$M_\nu \approx \frac{m_1}{6} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{\tilde{m}_2}{3} \begin{pmatrix} 1 + 2\sigma & 1 + \frac{1}{2}\sigma & 1 + \frac{1}{2}\sigma \\ & 1 - \sigma & 1 - \sigma \\ & & 1 - \sigma \end{pmatrix} + \frac{\tilde{m}_3}{2} \begin{pmatrix} 2\vartheta^2 & -\sqrt{2}\vartheta & \sqrt{2}\vartheta \\ & 1 - \vartheta^2 & -1 + \vartheta^2 \\ & & 1 - \vartheta^2 \end{pmatrix} \quad (7)$$

where $\vartheta = s_{13} e^{i\delta}$ with a Dirac phase δ .

The mass ratio m_1/m_2 is assumed to be much smaller than $m_2/m_3 = 0.17$. From the range of $|U|$ in Eq.(2), it is reasonable to estimate the upper bound of the σ and s_{13} to be about m_2/m_3 . Since m_1/m_2 is negligible in comparison with other parameters, the dimensionless matrix obtained by a denominator $m_3/2$ reduces to an expression with three leading parts depending on the orders and the types of parameters,

$$\begin{aligned} \frac{M_\nu}{m_3/2} = & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \frac{2m_2}{3m_3} e^{i\tilde{\varphi}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \sqrt{2}s_{13}e^{i\delta} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\ & + \mathcal{O}\left(\frac{m_1}{m_3}, \frac{m_2}{m_3}\sigma, s_{13}^2, \text{etc.}\right) \end{aligned} \quad (8)$$

where $\tilde{\varphi} = \varphi - \varphi'$. The entries in the three matrix terms represent exact ones or exact zeros while small contributions are all collected in the last term. The third term that represents the contribution of the small mixing angle $s_{13} \equiv \sin\theta_{13}$ is taken as not smaller than the ratio m_1/m_3 and other parameters mentioned in the fourth term. The value of m_2/m_3 is comparable to the upper bound of experimental value of $\sin\theta_{13}$. If some theoretical approaches or future experiments indicate the common size of the two values, the constraint of the large mixing angle θ_{12} implies that a difference between the Majorana phases and the Dirac phase $\varphi - \varphi' - \delta$ should be about $\pi/2$ to prevent the 1-2 and 1-3 matrix elements from the suppression by eliminating each other in two terms in Eq.(8). Since the order of the third term with s_{13} cannot exceed that of the second term with m_2/m_3 in any circumstance, the matrix M_ν/m_3 consists of mainly two scales of elements, $\mathcal{O}(1)$ and $\mathcal{O}(m_2/m_3)$.

III. A MODEL WITH $U(1) \times Z_3 \times Z_2$

The Yukawa interaction of leptons and the lepton number violating mass term of right-handed neutrinos

$$- \mathcal{L} = H\mathcal{Y}_\ell L_e \bar{e}_R + H\mathcal{Y}_\nu L_e \bar{N}_R + \frac{1}{2}M_R N_R N_R \quad (9)$$

consist of 3×3 matrices $\mathcal{Y}_\ell, \mathcal{Y}_\nu$ and M_R , where there are three right-handed neutrinos. The FN mechanism[8] is performed to generate the above matrices hereafter. In models whose flavor symmetry contains three distinct components $U(1) \times Z_3 \times Z_2$, we introduce three singlet scalars, S_0, S_1 and S_2 , with flavor charges

$$S_0(-1, 0, 0), \quad S_1(0, -1, 0), \quad S_2(0, 0, -1), \quad (10)$$

where the three elements in a parenthesis are three quantum numbers of each field under $U(1)$, Z_3 , and Z_2 , respectively. Let the Higgs be neutral under the flavor symmetry henceforth. The charges of fermions under the symmetry $U(1) \times Z_3 \times Z_2$ are denoted as

$$L_e(p_i^l, q_i^l, r_i^l), \quad N_R(p_i^r, q_i^r, r_i^r), \quad e_R(p_i^e, q_i^e, r_i^e), \quad (11)$$

where i denotes a generation. Then, the contributions to the Yukawa matrices arise from flavor invariant terms in

$$L_i \bar{e}_j H \left(\frac{S_0}{\Lambda_L} \right)^{p_{ij}^l} \left(\frac{S_1}{\Lambda_L} \right)^{q_{ij}^l} \left(\frac{S_2}{\Lambda_L} \right)^{r_{ij}^l} + L_i \bar{N}_j H \left(\frac{S_0}{\Lambda_L} \right)^{p_{ij}^n} \left(\frac{S_1}{\Lambda_L} \right)^{q_{ij}^n} \left(\frac{S_2}{\Lambda_L} \right)^{r_{ij}^n}, \quad (12)$$

where the exponents to the scalar fields S_0, S_1 , and S_2 are given with sums of the charges of fermions in Eq.(11). The sums p_{ij}^a, q_{ij}^a , and r_{ij}^a are calculated as follows[12][13]: A sum of $U(1)$ charges p_{ij}^a is obtained straightforwardly, $p_{ij}^l = p_i^l + p_j^e, p_{ij}^n = p_i^l + p_j^r$. On the other hand, a sum of Z_n charges q_{ij}^a or r_{ij}^a needs to be taken extra care of for the property of discrete abelian symmetry. If $q_i^l + q_j^e$ is less than 3, $q_{ij}^l = q_i^l + q_j^e$. If $q_i^l + q_j^e$ is equal to or larger than 3, $q_{ij}^l = q_i^l + q_j^e - 3$, that is, the charges are modded out by 3 and so are the q_{ij}^e and the q_{ij}^n . The particular rule for the charges of Z_3 symmetry is denoted by brackets $[\]_3$ such as $q_{ij}^l = [q_i^l + q_j^e]_3$ and $q_{ij}^n = [q_i^l + q_j^r]_3$. Likewise, $r_{ij}^l = [r_i^l + r_j^e]_2$ and $r_{ij}^n = [r_i^l + r_j^r]_2$. Also, the mass matrix M_R of the flavor invariant right-handed neutrino mass term is replaced by couplings including singlet particles,

$$\frac{1}{2} \Lambda_R N_R N_R \left(\frac{S_0}{\Lambda_L} \right)^{p_{ij}^r} \left(\frac{S_1}{\Lambda_L} \right)^{q_{ij}^r} \left(\frac{S_2}{\Lambda_L} \right)^{r_{ij}^r}, \quad (13)$$

where $p_{ij}^r = p_i^r + p_j^r, q_{ij}^r = [q_i^r + q_j^r]_3$, and $r_{ij}^r = [r_i^r + r_j^r]_2$.

The scalar fields in general can have different vacuum expectation values $\langle S_0 \rangle, \langle S_1 \rangle$ and $\langle S_2 \rangle$. These can be related to a common expansion parameter λ by setting

$$\frac{\langle S_0 \rangle}{\Lambda_L} \equiv \lambda, \quad \frac{\langle S_1 \rangle}{\Lambda_L} \equiv \lambda^\alpha, \quad \frac{\langle S_2 \rangle}{\Lambda_L} \equiv \lambda^\beta. \quad (14)$$

In general, one may take $\alpha \neq \beta$, but for our particular models we assume $\alpha = \beta$. The scale Λ_L where massive states are integrated out of the fundamental theory to produce an effective theory, is assumed to be larger than the vev $\langle S_0 \rangle$ of the singlet scalar field so that the parameter $\langle S_0 \rangle / \Lambda_L$ is smaller than one and is called λ . Then the generated terms in charged lepton Yukawa matrix will be of order $\lambda^{p_{ij}^l + \alpha(q_{ij}^l + r_{ij}^l)}$. We will restrict our attention to flavor charges that are non-negative. Even though only the standard model

fields plus right neutrinos are concerned here, the non-negativeness is motivated by analytic superpotential whose terms carry charges $p_{ij}, q_{ij}, r_{ij} \geq 0$. The lagrangian with broken flavor symmetry reduces to

$$-\mathcal{L} = HL_e \bar{e}_R \lambda^{p_{ij}^l + \alpha(q_{ij}^l + r_{ij}^l)} + HL_e \bar{N}_R \lambda^{p_{ij}^n + \alpha(q_{ij}^n + r_{ij}^n)} + \frac{1}{2} \Lambda_R N_R N_R \lambda^{p_{ij}^r + \alpha(q_{ij}^r + r_{ij}^r)} \quad (15)$$

There are undetermined order one coefficients multiplying these terms, and we assume that those coefficients are sufficiently close to one so as not to influence the hierarchy, i.e. somewhat greater than λ and somewhat less than $1/\lambda$.

The flavor charges (p_i, q_i, r_i) 's of i -generation fields are assigned as follows, if there are three families of right-handed neutrinos, $n_r = 3$;

$$\begin{array}{ccccc} & i=1 & i=2 & i=3 & \\ L_e & (p^l, q^l, r^l) : & (0, 2, 0) & (0, 0, 1) & (0, 0, 1) \\ N_R & (p^r, q^r, r^r) : & (2, 2, 1) & (0, 0, 0) & (0, 0, 0) \\ e_R & (p^e, q^e, r^e) : & (3, 0, 0) & (2, 0, 1) & (0, 0, 1) \end{array} \quad (16)$$

The mass matrix of right-handed neutrinos constructed by applying the flavor charges to Eq.(15) is

$$M_R \sim \begin{pmatrix} \lambda^{4+\alpha} & \lambda^{2+3\alpha} & \lambda^{2+3\alpha} \\ \lambda^{2+3\alpha} & 1 & 1 \\ \lambda^{2+3\alpha} & 1 & 1 \end{pmatrix} \Lambda_R, \quad (17)$$

and the Yukawa matrices of neutrinos and charged leptons are

$$\mathcal{Y}_\nu \sim \begin{pmatrix} \lambda^{2+2\alpha} & \lambda^{2\alpha} & \lambda^{2\alpha} \\ \lambda^{2+2\alpha} & \lambda^\alpha & \lambda^\alpha \\ \lambda^{2+2\alpha} & \rho \lambda^\alpha & \lambda^\alpha \end{pmatrix}, \quad \mathcal{Y}_\ell \sim \begin{pmatrix} \lambda^{3+2\alpha} & \lambda^{2+3\alpha} & \lambda^{3\alpha} \\ \lambda^{3+\alpha} & \lambda^2 & 1 \\ \lambda^{3+\alpha} & \lambda^2 & 1 \end{pmatrix}, \quad (18)$$

where the $\rho = 1$ in \mathcal{Y}_ν , but later the ρ will have different magnitude other than 1 in the basis of M_R diagonal. The exponents to λ in the above matrices are controlled by sum rules. The p_{ij} the sum of the $U(1)$ charges has a rule among themselves, $p_{ii} + p_{jj} - p_{ij} - p_{ji} = 0$, for $i \neq j$. Meanwhile, for the q_{ij} the modulated sum of the Z_3 charges, $q_{ii} + q_{jj} - q_{ij} - q_{ji}$ does not always vanish but can reduce to ± 3 . Likewise, $r_{ii} + r_{jj} - r_{ij} - r_{ji}$ does reduce to either 0 or ± 2 [13]. If one employs just a continuous $U(1)$ flavor symmetry, the type of neutrino mass matrix obtained via seesaw mechanism is democratic, which is caused by a feature of $U(1)$, the vanishing sum rule. The suppressed elements in the neutrino mass matrix can be derived using a discrete symmetry, if the charges of fields result in non-zero sum rule.

IV. MASSES OF LIGHT NEUTRINOS

The light neutrino masses are generated via the seesaw mechanism $M_\nu = \mathcal{Y}_\nu M_R^{-1} \mathcal{Y}_\nu^T$. Now on, the basis is switched to the one with right-handed neutrino mass eigenstates corresponding to mass eigenvalues (M_1, M_2, M_3) . The transformation of right-handed neutrinos may give rise to a change in the Yukawa matrix \mathcal{Y}_ν , which effect can appear as the suppression by ρ factor smaller than 1. Other effect than the ρ factor from the transformation of basis is nothing but rotations with angles about $\lambda^{2+3\alpha}$ which is negligible. Such a transformation does not change the order of magnitude of other contributions in \mathcal{Y}_ν . The light neutrino mass is obtained as

$$M_\nu \sim \frac{v^2 \lambda^{2\alpha}}{M_3} \begin{pmatrix} \lambda^{2\alpha} & \lambda^\alpha & \lambda^\alpha \\ \lambda^\alpha & 1 & 1 \\ \lambda^\alpha & 1 & 1 \end{pmatrix} + \frac{v^2 \lambda^{2\alpha}}{M_2} \begin{pmatrix} \lambda^{2\alpha} & \lambda^\alpha & \lambda^\alpha \\ \lambda^\alpha & 1 & \rho \\ \lambda^\alpha & \rho & \rho^2 \end{pmatrix} + \frac{v^2 \lambda^{4+4\alpha}}{M_1} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (19)$$

where v is the vacuum expectation value of the doublet Higgs.

As shown in Eq.(16), there is no distinguishability between the second and the third neutrinos under the flavor symmetry. The right-handed neutrinos described in Eq.(17) may have two degenerate masses, $M_2 = M_3$, or may have three different masses with maximal 2-3 mixing angle. If $M_2 < M_3$, the second term in Eq.(19) will exceed the first term, which is not the case to be concerned in this model. The leading orders in the elements of neutrino mass matrix can be arranged in three parts, with $M_1 \sim \lambda^{4+\alpha} \Lambda_R$ and $M_2 \sim M_3 \sim \Lambda_R$,

$$M_\nu \sim \frac{v^2 \lambda^{2\alpha}}{\Lambda_R} \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \lambda^\alpha \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \lambda^\alpha \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^{2\alpha}) \right\}, \quad (20)$$

which shows that the suppression due to the change of the basis does not affect the order of those three terms. Thus, the masses of the light neutrinos are predicted only in terms of the scale of lepton number violation Λ_R and the breaking scale of the discrete flavor symmetry $Z_3 \times Z_2$, $\lambda^\alpha \equiv \langle S \rangle / M_\Lambda$. The breaking scale of discrete symmetries may be the same as that of the $U(1)$ flavor symmetry, $\alpha = 1$, or may not. But, before removing α which denoted the contribution of discrete symmetry with λ^α , it is worth stressing that the particular structure of the matrix in Eq.(20) was able to be derived only due to the discrete

symmetries. If $\alpha = 1$, the model predicts in comparison of Eq.(8) and Eq.(20)

$$\frac{2}{3} \frac{m_2}{m_3} \sim \sqrt{2} \sin \theta_{13} \sim \lambda, \quad (21)$$

and for charged leptons with the Yukawa matrix in Eq.(18)

$$\frac{m_e}{m_\tau} \sim \lambda^5, \quad \frac{m_\mu}{m_\tau} \sim \lambda^2, \quad (22)$$

which are consistent to each other as well as experiments, while they are derived in terms of a single parameter λ induced from the discrete symmetry breaking. Two relations in Eq.(22) give rise to estimation of λ about 0.19 and about 0.24, respectively. They would provide an appropriate range for values in Eq.(21).

V. REMARKS

It was shown that a $U(1) \times Z_3 \times Z_2$ flavor symmetry invariant model of Yukawa interaction gives rise to the prediction of Eq.(21) and Eq.(22) through a series of mechanisms of breaking the symmetry and the seesaw. The above prediction is obtained with Yukawa matrices at lepton number violating scale M_1 or $M_3(\Lambda_R)$. The test of $\sin \theta_{13}$ with future experiments requires a detailed model that involves the choice of order one coefficients and running of Yukawa couplings. With respect to RG running, the mass ratios mentioned in Eq.(21) and in Eq.(22) are stable in Standard Model or in Minimal Supersymmetric Standard Model (MSSM) [15][16]. However the running of a neutrino mixing angle like θ_{13} in Eq.(21) reveals different aspects depending on models. It was pointed out in Ref.[17] that the RG evolution of θ_{13} may be positive or negative relying on whether a model in MSSM includes CP phases or not.

One can recognize that the model obtains the large mixing between the second and the third generations simultaneously both in the Yukawa matrix of charged leptons \mathcal{Y}_ℓ in Eq.(18) and in the neutrino mass matrix. The MNS mixing matrix, the multiplication of the transposed mixing matrix of charged leptons and the mixing matrix of neutrinos[5], can maintain still large or maximal mixing due to imaginary phases even if both mixing matrices have large or maximal angles for the relevant mixing before the multiplication.

Under the flavor symmetry, only the first generation of neutrinos can be distinguished from others. Other two generations of neutrinos cannot be distinguished from each other

in terms of their charges no matter which are left-handed or right-handed, although three generations of the left-handed are classified within weak interaction. With the development to this point, there is no way to state that the right-handed are of two generations or they are of three generations. Some phenomenology with righthanded neutrinos, e.g., leptogenesis, may help further setting of the model.

In the model derived by FN mechanism extended with additional discrete symmetries, it was shown that the small but moderate neutrino mass ratio m_2/m_3 is a consequence from breaking the discrete symmetries. The small value for θ_{13} is also explained as another consequence of discrete symmetry breaking, which is not plausible only by a continuous U(1) symmetry.

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- [1] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81**, 1562 (1998)
S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **86**, 5651 (2001)
 - [2] Q. Ahmad *et al.* [SNO Collab.], Phys. Rev. Lett. **87**, 071301 (2001); Phys. Rev. Lett. **89**, 011301 (2002);
 - [3] M. Apollonio *et al.* [CHOOZ Collaboration], Phys. Lett. B **420** (1998) 397.
 - [4] K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90**, 021802 (2003)
 - [5] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
 - [6] W. L. Guo and Z. Z. Xing, Phys. Rev. D **67**, 053002 (2003) M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Lett. B **562**, 273 (2003) M. C. Gonzalez-Garcia and C. Pena-Garay, Phys. Rev. D **68**, 093003 (2003) M. Maltoni, T. Schwetz, M.A.Tortola and J. W. F. Valle, Phys. Rev. **D68**, 113010 (2003).
 - [7] P.F.Harrison, D.H.Perkins, W.G.Scott, Phys. Lett. **B 458** (1999) 79; Phys. Lett. **B 530** (2002) 167; Z.-z. Xing, Phys. Lett. **B 533** (2002) 85; C. S. Lam, Phys. Lett. **B 507** (2002) 214; X.G. He and A. Zee, Phys. Lett. **B 560** (2003) 87; Phys. Rev. **D 68** (2003) 037302 ; Y. Koide, Phys. Lett. **B 574** (2003) 82; S. h. Chang, S. K. Kang and K. Siyeon, Phys. Lett. B **597**, 78

- (2004)
- [8] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147**, 277 (1979)
 - [9] M. Gell-Mann, P. Ramond and R. Slansky, Print-80-0576 (CERN). T. Yanagida, Proceedings Of The Workshop On The Unified Theory And The Baryon Number In The Universe, R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980)
 - [10] L. Hall, H. Murayama and N. Weiner, Phys. Rev. Lett. **84**, 2572 (2000) N. Haba and H. Murayama, Phys. Rev. D **63**, 053010 (2001). M. S. Berger and K. Siyeon, Phys. Rev. D **63**, 057302 (2001)
 - [11] F. S. Ling and P. Ramond, Phys. Lett. B **543**, 29 (2002) G. Altarelli and F. Feruglio, New J. Phys. **6**, 106 (2004)
 - [12] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **398**, 319 (1993); Nucl. Phys. B **420**, 468 (1994) Y. Grossman, Y. Nir and Y. Shadmi, JHEP **9810**, 007 (1998) Y. Nir and Y. Shadmi, JHEP **9905**, 023 (1999)
 - [13] M. S. Berger and K. Siyeon, Phys. Rev. D **62**, 033004 (2000); Phys. Rev. D **64**, 053006 (2001)
 - [14] M. Tanimoto, Phys. Lett. B **456**, 220 (1999) W. Grimus, A. S. Joshipura, L. Lavoura and M. Tanimoto, Eur. Phys. J. C **36**, 227 (2004)
 - [15] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998)
 - [16] Y. Koide and H. Fusaoka, Phys. Rev. D **64**, 053014 (2001)
 - [17] S. Antusch, J. Kersten, M. Lindner and M. Ratz, Nucl. Phys. B **674**, 401 (2003)